AD-603 929

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C. H. Mok and J. Duffy

Technical Report No. Nonr-562(20)/
Division of Engineering
Brown University
Providence, R. I. 02912

June, 1964

Sponsored by Dept. of the Navy
Office of Naval Research
Under Contract Nonr-562(20)37
Task Order NR-064-424

ACKNOWLEDGMENTS

The helpful discussions with Professor D. C. Drucker, Professor H. Kolsky and Professor P. S. Symonds are greatly appreciated

Thanks are also due to Messrs. R. Pagliarini, P. Rush and R. R. Stanton for their assistance in the experimental work and to Mrs. J. Alexander and Mr. A. Abols for preparing the manuscript and figures.

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Abstract

An analysis is presented of the impact process occuring when a hard ball strikes the surface of a large metal specimen. It is shown that the results of dynamic indentation tests can be interpreted to provide a measure of the yield stress of the metal over a range of strains taking into account the influence of strain-rate. Experiments are described in which steel balls are allowed to drop onto specimens of lead, steel and aluminum. In each test the velocities of impact and rebound are measured as well as the diameter of the indentation remaining in the metal specimen and the time of contact between specimen and indenter. These experimental results are used in conjunction with the analysis to provide for each metal a dynamic stress-strain curve which is then compared to results of a simple compression test performed under impact conditions. For all metals the best agreement occurs if the simple compression tests are performed at a strain-rate of 1,500 sec^{-1}. Finally, the present analysis provides a model of the indentation process which furnishes quite accurate predictions of the size of the permanent indentation, the time of contact during impact and the coefficient of restitution, if the dynamic stress-strain relation of the material is known.

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* The results in this paper were obtained in the course of research sponsored by the Office of Naval Research under Contract Nonr-562(20) with Brown University

** Respectively Research Assistant and Associate Professor, Division of Engineering, Brown University, Providence, R. I.

1. Introduction.

In one of the simplest of dynamic tests a small but hard indenter falls under the action of gravity onto a massive specimen. It is an easy matter, in general, to measure the velocities of impact and of rebound and the size of the remaining indentation in the specimen. In the past this test has received considerable attention from numerous investigators [1-4]*. Unfortunately, there is no completely satisfactory analysis of the elastic-plastic problem so that it is extremely difficult to use this indentation test as a means of determining material properties. Martel [5] suggested that the yield pressure be treated as constant from the start to the end of impact, and Tabor's work [1] is based largely on this assumption for which there is experimental evidence in the work of Crook [6] and of Tabor himself. The recent work of Goldsmith et al. [7 and 8] as well as the results of a previous paper [9] seem to indicate that the traditional concept of a constant yield pressure during impact is not adequate. Furthermore, this concept is not consistent with the results obtained in dynamic tests of an entirely different nature performed by other investigators. The present investigation is intended to examine the feasibility of using a simple impact test to determine the dependence of yield stress on strain and on strain-rate.

To achieve this aim, it is necessary first that an expression be established for the yield pressure existing underneath the ball and that, in turn, yield pressure be related to yield stress. Secondly, it is necessary that the strain and the strain-rate be evaluated using the diameter of the permanent indentation in the specimen and the time of contact during impact. Tabor and Martel have given expressions for the yield pressure in terms of measurable quantities [1]. However, in deriving both of these expressions the yield pressure is taken as constant, whereas to investigate the dependence of yield

*Numbers in square brackets refer to the Bibliography.

stress on strain and on strain-rate, work-hardening must be taken into account. Therefore, a new expression for yield pressure is needed, Equation (7). It is derived using conservation of energy and an empirical power relation between impact velocity and indentation diameter. Such a power relation apparently holds very closely for a number of metals and alloys over a wide range of temperatures [9 and 10]. Finally, the yield stress and the yield strain are found using the empirical formulae

$$\sigma = P/c \qquad (1)$$

$$\varepsilon = d/5D \qquad (2)$$

first proposed by Tabor and in which where c is a constant near to 2.8 and σ is the true stress and ε the true strain. These formulae are based on a comparison of the results of static indentation tests and simple compression tests. It is felt they can be applied in the present experiments because, as explained below, the impact velocities are not so high that inertia effects become important.

It is comforting to know that the expression for yield pressure mentioned above and presented in Equation (7) can be derived in a completely different manner by taking for the relation between stress and strain in the specimen an expression of the type $\sigma = b\varepsilon^n$. Finally, by combining this with Newton's second law, an expression is obtained containing the parameters b and n which predicts quite accurately the size of the permanent indentation in the metal specimen.

The results of the analysis were examined by performing impact tests with steel balls on four different metals and plotting the computed yield stress against strain. For comparison a series of dynamic tests in simple compression were made on the same metals keeping the strain-rate as nearly

constant as possible in each test. The two sets of results compare favorably, indicating that within the limits of the present experiments the dynamic yield stress of metals can be measured in impact tests with a hard ball. The results also show that the yield stress of lead and of steel depend more strongly on strain-rate than that of the aluminum alloys tested.

II. Analysis of the Indentation Process under Impact

The impact velocity in the present tests is sufficiently low so that for purposes of analysis the impact process can be taken as quasi-static. This means that within and in the neighborhood of the indentation the strain distribution (although not the strain magnitude) is independent of impact velocity. There is considerable evidence to suggest that this gives a good description of impact within the range of present velocities, 1 cm/sec-600 cm/sec. (See Goldsmith et al. [8] and Hunter [11].)

One can simplify the problem further, in view of the pronounced plastic flow during impact, by neglecting the initial elastic stage and thus considering only a two-stage impact process in which the initial plastic work-hardening is followed by an elastic recovery once the whole forward energy of the indenting ball has been consumed. In this it is presumed that the ball itself undergoes no plastic deformation at any stage during impact. It is true, as pointed out by K. L. Johnson [12], that plastic effects are present in the specimen during recovery but these are neglected in the analysis. There remains still the question of the influence of strain-rate on the material properties of the specimen. Obviously, the strain-rate changes during each impact test starting at a value which probably depends on the initial velocity of the ball and going down to zero. It is most likely also that the strain-rate varies from test to test. However, for the present analysis each impact is presumed to have one characteristic or average strain-rate. This is clearly

a strong simplification, but it is a necessary one until a better analysis of the strain distribution is available. The justification for adopting this simplification depends in part on the results. As will be seen, the present analysis when applied to test results, leads to dynamic stress-strain curves corresponding to a constant strain-rate which are in good agreement with those of other investigators.

(1) Plastic Strain-Hardening Stage

Impact is presumed to start with a plastic strain-hardening stage in which the motion of the ball is resisted by the specimen material with a pressure dependent on its work-hardening properties. The expression for yield pressure is derived from impact test results and using conservation of energy. It is shown later that this derivation is equivalent to making use of a stress-strain relation of the type $\sigma = b\varepsilon^n$ for the plastic loading stage.

After the ball strikes the specimen, the rate of loss of kinetic energy T which it suffers must at each instant equal the rate of work of the indenting force, to a close enough approximation. If x is the approach of the two bodies, then

$$(P\pi a^2/4)\ \delta x = \delta T \tag{3}$$

where a is the diameter of the indentation at a given x. For a spherical ball striking a plane surface

$$\delta a/\delta x = 2D/a \tag{4}$$

where D is the ball diameter. In writing this expression, it is presumed that the indenting ball is hard and that there is not too much "piling-up" or "sinking-in" near the contact area. Combining Equations (3) and (4) gives

$$P = (8D/\pi a^3)(\delta T/\delta a) \tag{5}$$

for the yield pressure. It remains to evaluate $\delta T/\delta a$.

Previous results indicate that the diameter of the permanent indentation d produced by impact at a velocity v_1 is proportional to $v_1^{1/\alpha}$ where α is a constant [9]. This relation holds quite closely for the two materials tested with α lying between 0.44 and 0.47 irrespective of temperature (see also Figures 9 and 10). Hence, the energy W lost to deformation must be proportional to $d^{2/\alpha}$. If, furthermore, the indentation process occurs quasi-statically and if the strain-rate does not influence strongly the relation between load and deflection, then the energy lost at any stage of impact will depend only on the depth of penetration. This means that throughout the impact process W is proportional to $a^{2/\alpha}$, so that $\delta W/\delta a = 2W/\alpha a$. Applying the principle of the conservation of energy $\delta W = \delta T$ so that

$$P = \left(\frac{1}{2\alpha}\right) W/(\pi a^4/32D) \tag{6}$$

at any stage of impact. The maximum value of P occurs when a becomes equal to d and is given by

$$P = \left(\frac{1}{2\alpha}\right)\left(\frac{1}{2} mv_1^2\right)/(\pi d^4/32D) \ . \tag{7}$$

This is the expression for the yield pressure used in the present investigation. It may be compared to

$$(P)_M = \frac{1}{2} mv_1^2 \ /(\pi d^4/32D) \tag{8}$$

derived by Martel and

$$(P)_T = \left(1 - \frac{3}{8} e^2\right)\left(\frac{1}{2} mv_1^2\right)/(\pi d^4/32D) \tag{9}$$

derived by Tabor, in which e is the coefficient of restitution. The only difference between Equation (7) and Equations (8) and (9) is that (7) takes into account the work-hardening properties of the material where (8) and (9) are derived for a constant yield pressure. $(P)_M$ must be greater than $(P)_T$ and experimental results indicate that, in general, P is greater than $(P)_M$.

As mentioned above, an alternative derivation is available leading again to the expression for the yield pressure in Equation (7). This derivation makes use of Newton's second law and the stress-strain relation $\sigma = b\epsilon^n$ applied to the plastic loading stage. One must, in addition, employ Tabor's empirical formulae, Equations (1) and (2), so that effectively one again takes the impact process as quasi-static. The result of the two derivations is the same if $2/(4 + n) = \alpha$. In the analysis, as shown below in Equation (10), $2/(4 + n)$ is the exponent of the theoretical power relation between impact velocity and indentation diameter derived from the above stress-strain relation and Newton's law. Consequently, $2/(4 + n)$ is equivalent in meaning to the parameter α which is also the exponent of a power relation between impact velocity and indentation diameter. Looking at experimental results, α is found from the slope of the experimental $\log d - \log v_i$ curve (Figures 9 and 10) obtained in the impact tests with a hard ball, whereas n depends on the strain-rate with which one runs the simple compression test to determine the stress-strain curve for the material. It turns out that $\alpha = 2/(4 + n)$ very closely for all four metals if the strain-rate is set approximately at 1,500 sec^{-1}. This question is discussed further in Section IV, paragraph 2a.

Once the yield pressure is determined from the results of indentation tests, the stress-strain relation of a metal can be found using Tabor's empirical formulae, Equations (1) and (2), and the diameter of the permanent indentation. Inversely, as shown below, if the stress-strain relation can be approximated by a curve of the form $\sigma = \epsilon^n$, then it is possible to predict the indentation diameter, the coefficient of restitution and the time of contact of the dynamic indentation test. Applying Newton's law to describe the motion of the ball during the plastic stage (Appendix B), the diameter of the contact

area at maximum relative approach is found to be

$$d = f_1(c,n,b) \quad D\,(mv_1^2/D^3)^{1/(4+n)} \tag{10}$$

where

$$f_1(c,n,b) = [5^n \cdot 4 \cdot (4 + n)/\pi cb]^{1/(4+n)}$$

and m is the mass of the ball. It is reasonable to suppose that the diameter of the maximum contact area is approximately equal to that of the permanent indentation, as long as plastic deformation is pronounced and the elastic recovery relatively small. One should compare Equation (10) to the expression for d/D derived using dimensional analysis in Reference [9]. Both equations predict that d/D will be proportional to $(mv_1^2/D^3)^{\alpha/2}$ if, as mentioned above, α equals $2/(4 + n)$. The dependence on yield stress comes into the function $f_1(c,n,b)$ in Equation (10). It is not clear how strong is the influence of the Young's modulus of the specimen, presumably it also influences $f_1(c,n,b)$.

For values of n between 0 and 1 it is shown in Appendix B that the duration of the plastic strain-hardening stage (the time lapse between the instant when the impact starts and the instant when the maximum relative approach is attained) is

$$t_p = \frac{\pi}{8}\,[f_1(c,n,b)]^2 \quad (D/v_1)(mv_1^2/D^3)^{2/(4+n)} \quad . \tag{11}$$

However, to obtain the total time of contact and the coefficient of restitution one must consider also the recovery stage.

(2) Elastic Recovery Stage

The coefficient of restitution and the time of contact are obtained by taking the recovery stage as entirely elastic and using Hertz's theory of impact between elastic bodies [13]. As Tabor [1] indicated, recovery can be treated as the reverse in time of the impact between an elastic ball of

radius $r_1 = D/2$ moving at the rebound velocity v_2 and striking an elastic spherical seat of radius r_2 , where r_2 is the radius of curvature of the permanent indentation in the specimen. For purposes of analysis the initial diameter of the contact area during recovery is taken equal to the contact diameter occurring at the end of the plastic stage. Moreover, if the force at the start of recovery is obtained from a stress-strain relation of the type $\sigma = B\varepsilon^N$, where B and N are constants, it can be shown using Tabor's empirical formulae, Equations (1) and (2), that the coefficient of restitution is given by

$$e = v_2/v_1$$

$$= 1.1 \; K \; L \; (mv_1^2/D^3)^{(2N-n-1)/2(4+n)} \qquad (12)$$

where

$$L = (4 + N)[f_1(c,n,b)]^{(3+2N)/2}/[f_1(c,N,B)]^{4+N}$$

and

$$K = [(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2]^{-1/2}$$

in which E_1 , ν_1 and E_2 , ν_2 are, respectively, the Young's moduli and Poisson's ratios of ball and specimen. From Hertz's theory the duration of the recovery stage is given by

$$t_e = 2.02 \; K \; [f_1(c,n,b)]^{-1/2} \; (m/D)^{1/2} \; (mv_1^2/D^3)^{-1/2(4+n)} \qquad (13)$$

and if $0 \leq n \leq 1$ the total time of contact during impact, t , is given approximately by

$$t = G \; (m/D)^{1/2} \; (mv_1^2/D^3)^{-(n+1)/4(4+n)} \qquad (14)$$

where

$$G = 2.02\ K\ (H + 1)/[f_1(c,n,b)]^{1/2}\ (mv_1^2/D^3)^{(1-n)/4(4+n)}$$

and

$$H = \frac{\pi}{16.2\ K}\ [f_1(c,n,b)]^{5/2}\ (mv_1^2/D^3)^{(1-n)/2(4+n)}\ .$$

Experimentally, Tabor found that the magnitude of the yield pressure at the beginning of elastic recovery is in general lower than that of the mean yield pressure during the plastic stage and is closer to the static yield pressure needed to produce an indentation of the same size. This is the reason that the static relation $\sigma = B\varepsilon^N$ is used for the recovery while the dynamic relation $\sigma = b\varepsilon^n$ is used for loading.

In summing up, the present analysis provides a method for determining the dynamic stress-strain relation of a metal from the results of dynamic indentation tests. Inversely, if the material properties are known, one can predict the size of the indentation, the coefficient of restitution and the time of contact during impact. The experiments described below are intended to demonstrate the reliability of the method.

III <u>Experimental Work</u>

Specimens of four different materials were tested at room temperature (Table 1). As explained above, the purpose is to find a method which can be used to predict the dynamic yield stress of a material at a given strain and a given strain-rate from the results of dynamic indentation tests. For comparison, stress-strain curves were obtained in more conventional simple compression tests at various constant strain-rates. However, to check the applicability of Tabor's empirical formulae, indentation and simple compression tests were also performed under static conditions.

(1) Static Indentation Tests

polished surface of a 1 1/2 in. thick specimen. In each test the maximum load, F , was recorded and the diameter of the permanent indentation, d , was obtained by averaging four measurements made with a microscope. The pressing speed was held constant and chosen to keep the quantity d/5tD approximately equal to 0.001 sec^{-1} , where t is the total time of loading measured with a stop watch. Results are presented in Figure 2, and in Appendix C.

(2) Dynamic Indentation Tests

The experimental technique is the same as that described in Reference [9] to which the reader is referred for details. The specimens were again of 6 in. dia. and 10 in. in length, but only steel balls were used as indenters In each test, impact velocity , indentation diameter, coefficient of restitution and time of contact during impact were measured. Results are presented in Figures 9 to 12 and in Appendix C.

(3) Static Tests in Simple Compression (Strain Rates = 0.001 sec^{-1})

Cylindrical specimens 1/4 in. long and of 1/4 in. dia. were compressed between two parallel surfaces of hardened steel lubricated with a graphite oil mixture. (For lead the specimens had a 3/8 in. dia.) To make sure that the influence of friction was negligible, a few specimens of greater length were tested and found to give the same results to within the estimated experimental accuracy. In each test the load and deformation were recorded continuously and the true stress and true strain computed taking the material as incompressible. Results for the four metals tested are presented as the solid lines in Figure 3.

(4) Dynamic Tests in Simple Compression

Circular cylindrical specimens of the same dimensions as those tested in the static compression tests were compressed against a Hopkinson bar by the impact of a carriage travelling at velocities between 100 and 1,000 cm/sec.

The method of Karman and Duwez [14] was adopted to keep the strain-rate as nearly constant as possible throughout the test. For this purpose the carriage was made very heavy (10 lbs) and a disk of brittle material placed in front of it which fractured shortly after impact and before the velocity of the carriage changed substantially (Figure 1). Besides holding the strain-rate nearly constant, this technique makes it possible to control fairly closely the total deformation in the specimen. Each test provided one point on the stress-strain curve, namely, the maximum strain and the presumed corresponding stress. The difference between the final and the initial lengths of the specimen gave the maximum strain, and the stress was found from the force pulse as measured with the Hopkinson bar (a slender bar with strain gages which was calibrated under both static and dynamic loading conditions). The average strain-rate was evaluated as the total strain divided by the total time of loading as measured on the force pulse record.

It is felt that the above set-up provided fairly closely the desired experimental conditions. A number of tests with specimens of various dimensions gave substantially the same results, so that the influence of waves within the specimen and of the end conditions probably was not great. Calculations based on Davies' results [15] indicated that the geometrical dispersion within the Hopkinson bar should not affect the measurement of stress. For a further critique of the technique the reader is referred to the work of Kolsky [16] and of Hunter and Davies[17].

Figure 3 presents the results of this experiment in a true stress-true strain plot. Dotted lines are drawn through the data points obtained for each material at each of two averaged strain-rates, namely ~ 150 and ~ 1,500 sec^{-1}. These results agree with the work by Lindholm [18], Manjoine [19], and Johnson et al. [20]. It can be seen that for a constant strain the yield stress

increases with strain-rate. This dependence on strain-rate is greater for lead and steel (approximately 100%) than it is for either of the two aluminum alloys (approximately 20%).

IV. Analysis of Test Results

(1) The Yield Stress as Found in Simple Compression and in Indentation Tests

(a) Static Yield Stress

The maximum yield pressure in the static indentation tests is found by dividing the maximum force, F, as given in Figure 2, by the area of the permanent indentation, $\pi d^2/4$. The stress and Strain are computed using Tabor's formulae, Equations (1) and (2), and the results compared to the static strain-stress curve obtained in simple compression (Figures 4-7). As may be seen from the figures, the ratio of the yield pressure obtained in indentation tests to the yield stress found in simple compression remains substantially constant. However, for three of the materials tested the value of this constant c is nearer 3.0 than it is to Tabor's 2.8, and in the case of lead has a value of 3.59 (Table 1). The reason that lead has a higher value of c is apparently due to a difference in its yield mechanism. Lead showed non-axial symmetric yielding both in the simple compression and in the indentation tests. In simple compression lead yielded along two shear planes oriented at about 45° to the axis of the cylindrical specimen, while the indentation produced by the ball appeared from above to be a square with rounded corners, (Figure 8). This appearance is due to an alternate "Piling-up" and "Sinking-in" of the material as one goes around the circumference of the indentation. In contrast with the behavior of lead both aluminum and steel deformed symmetrically. In aluminum in simple compression the slip lines on the wall of the cylindrical specimen are parallel to the axis. In steel they are helices. In the indentation

tests with aluminum and steel the indentation appeared circular. However, the slip lines for aluminum lie along a radius whereas they are spirals for the steel.

(b) Dynamic Yield Stress

The results of the dynamic indentation tests are interpreted in terms of a dynamic yield stress and a corresponding strain by using Equation (7) for the dynamic yield pressure and Tabor's formulae, Equations (1) and (2). In accordance with the analysis given above, the slope of the log v_1 - log d curve provides the necessary value of α in Equation (7), while the value of c in Tabor's formulae is taken as equal to that found statically (Table 1). The yield stress thus obtained is compared in Figures 4-7 with the results of the dynamic compression tests. For all four materials and irrespective of the indenter the best agreement is found with the stress-strain curve obtained in dynamic compression tests performed at a constant strain-rate of approximately 1,500 sec^{-1}. The reason for this is not clear. In view of the fairly wide range of impact velocities in the experiments (2 cm/sec to 600 cm/sec) and the corresponding change in the values of the average strain-rate as measured by d/5tD (about 20 times), one might believe that the strain-rate would vary considerably during a given test as well as from test to test. There are, however, two effects which when combined, might produce the present experimental result. The first is geometric. When a ball is pushed into the plane surface of a body, the volume of the indentation increases as the square of the depth of penetration. This means that the more significant part of the impact, in terms of metal deformed or energy dissipated, will come near the end of the plastic stage. Therefore, the properties of the metal near the end rather than at the start of impact will be dominant. In a low velocity impact the depth of penetration is small and this difference is not important, but in

impact at a high velocity the end of the plastic stage will be dominant and by then the ball is moving much more slowly and the strain-rate is decreased. Hence, the total (or average) response of the metal will not be influenced greatly by the high yield stresses produced by the large strain-rates at the start of impact. Therefore, the influence of impact velocity on the average yield stress is small. In addition to the effects of geometry, a second factor which influences results involves the variation of the yield stress with strain-rate. In general, the yield stress of metals and alloys does not change greatly with a change in the magnitude of the strain-rate. If then the precise value of the strain-rate is not important, then an average strain-rate becomes more meaningful and the indentation test provides a good measure of the dynamic yield stress.

Other investigators have obtained similar results as regards strain-rate. Goldsmith and Yew [21] measured the impact force as a conical indenter entered a specimen. They found that to a first approximation the dynamic force-indentation curves are independent of impact velocities ranging from 1,800 cm/sec to 10,500 cm/sec and that these curves always lie above the static force-indentation curves. A similar result was obtained by Goldsmith and Lyman [7] using balls as indenters.

As shown below, the diameter of the indentation is predicted quite accurately by employing a model consisting of a plastic work-hardening stage corresponding to a constant strain-rate of 1,500 $\sec^{-1}$. It is felt that this agreement provides further evidence tending to confirm the validity of the present analysis.

(2) Predictions of Indentation Diameter, Contact Time during Impact and Coefficient of Restitution

(a) Indention Diameter

The diameter of the indentation remaining in the specimen after imp at a given velocity is predicted above by Equation (10). The parameters b and n, which define the stress-strain relation of the material, are evaluated from the results of tests at constant strain-rates (Table 1). Predic values of the indentation diameter are plotted in Figures 9 and 10 for a ra of strain-rates and compared to experimental values. The best agreement fc all metals is found if one chooses a strain-rate of 1,500 sec^{-1}.

In addition to predicting correctly the size of the indentation, th analysis predicts quite accurately the dependence of the indentation diamet on the impact velocity. It will be recalled that according to experimental results, d is proportional to v_1^{α} where α is a constant independent o size of the ball [9]. If one uses for each metal the stress-strain curve corresponding to a strain-rate of 1,500 sec^{-1}, then Equation (10) predicts that α will have a value of 0.455, 0.460, 0.466, and 0.493, respectively, the lead, 6061 aluminum, 1100 aluminum and C1018 steel. Experimental result give for the same quantities values of 0.441, 0.456, 0.466, and 0.491, respectively, for the same metals. A perfectly plastic theory of impact pre dicts that α will equal 0.50 [9].

(b) **Impact Time**

The total time of contact during impact is predicted using Equation The parameters in this equation which depend on the properties of the specim i.e. b and n, were evaluated from the results of the simple compression tests run at a strain-rate of 1,500 sec^{-1}. A comparison with experimental results is presente i Figure 11 for the four materials tested. Agreement good except in the l r velocity range. This discrepancy may be due to the fact that the initial elastic stage is neglected in deriving Equation (14) an that this stage becomes more important at lower velocities.

For each material the numerical value of the function G in Equation (14) varies no more than 10% for the range of velocities from 2 to 600 cm/sec. As a result, for any of these materials t is approximately proportional to $(m/D)^{1/2} (mv_1^2/D^3)^{\beta/2}$ where $\beta = -(1 + n)/2(4 + n)$. From the results of simple compression tests, Table 1, the numerical value of β was found to lie between -0.13 and -0.16. Dimensional analysis [9] also indicates that t is proportional to the above factor and according to the results of the indentation tests β lies between -0.13 and -0.19 (including tests at elevated temperatures).

(c) Coefficient of Restitution

The coefficient of restitution as predicted using Equation (12) is compared to measured values in Figure 12. In the calculation of the coefficient of restitution values of b and n were obtained from the stress-strain curve corresponding to a strain-rate of 1,500 sec^{-1}, and those of B and N from the static curve (corresponding to a strain rate of 0.001 sec^{-1}). In addition, the values of E for the specimen and ball were taken from Table 1, whereas ν is always equal to 0.3. It is shown that over the range of velocities Equation (12) gives a good prediction of e for all materials tested except steel. In the case of steel, the error in the prediction is approximately 50%.

CONCLUSIONS

It is shown that the dynamic yield stress of a metal can be predicted over a range of strains through a proper interpretation of the results of indentation tests. Experiments performed on four metals give values of the yield stress which agree closely with the results of impact tests in simple compression. Thus in spite of the complexities of the indentation problem, it can give an accurate measure of the yield stress. This conclusion is important because the indentation test is easily performed even at elevated temperatures. At first sight, it is surprising that this degree of success is achieved in a test in which the results are sensitive only to averaged or integrated values of stress and of strain. The reason probably lies in the geometry of the colliding bodies combined with the fact that yield stress is not very sensitive to changes in strain-rate within the limits of the present experiment. This question is discussed further in the text. As far as actual values are concerned, it was found that the yield stress for lead and aluminum is approximately twice as great dynamically as statically over the range of strains; for aluminum the difference is only about 20%.

One can predict also the magnitude of the diameter of indentation, the total time of contact, and the coefficient of restitution using a simple model in which the specimen exhibits plastic work-hardening during loading and an elastic recovery. If the strain-rate during plastic work-hardening is taken at a constant 1,500 sec^{-1}, a very accurate prediction of the diameter of the indentation is obtained over the range of impact velocities. This is true of the four metals tested. As explained above, this agreement is probably due to geometry effects and to the manner in which yield stress depends on strain-rate. Instead of the model used in the present investigation, the yield stress could

be made to depend on the strain-rate and not on the strain. However, one can show that in predicting results such a model is far inferior to the strain-hardening one.

An earlier set of experiments indicated that for each metal a power relation exists between the diameter of indentation and the velocity of the ball at impact [9]. It is evident that this velocity-indentation relation must depend on the dynamic stress-strain relation of the metal. The present investigation shows that a work-hardening model and a stress-strain relation of the form $\sigma = b\varepsilon^n$ can be used to predict accurately the velocity-indentation relation, and furthermore that the material constants b and n determine the magnitudes of the parameters in this velocity-indentation relation.

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TABLE 1

Materials Tested

Values of b and n in $\sigma = b\varepsilon^n$ are obtained from Figure 3.

Specimen Material	Annealing	Young's Modulus, E 10^6psi	Young's Modulus, E 10^3kg/mm^2	Tabor's Constant c	Strain Rate (sec^{-1})	b 10^3psi	b kg/mm^2	n
lead	-	2.32	1.63	3.59	0.001	3.02	2.12	0.256
					150	7.80	5.50	0.432
					1500	8.42	5.93	0.397
aluminum (6061-T6)	900°F for 3 hrs.	10.0	7.14	3.02	0.001	51.4	36.2	0.360
					200-1500	60.0	42.3	0.347
steel (C1018)	1650°F for 2 hrs.	29.0	20.4	3.16	0.001	83.0	58.4	0.214
					150	78.0	55.0	0.0536
					1300	92.5	65.2	0.0536
aluminum (1100F)	620°F for 2 hrs.	10.0	7.14	2.95	0.001	24.3	17.1	0.264
					200	34.0	24.0	0.314
					1500	36.0	25.3	0.294

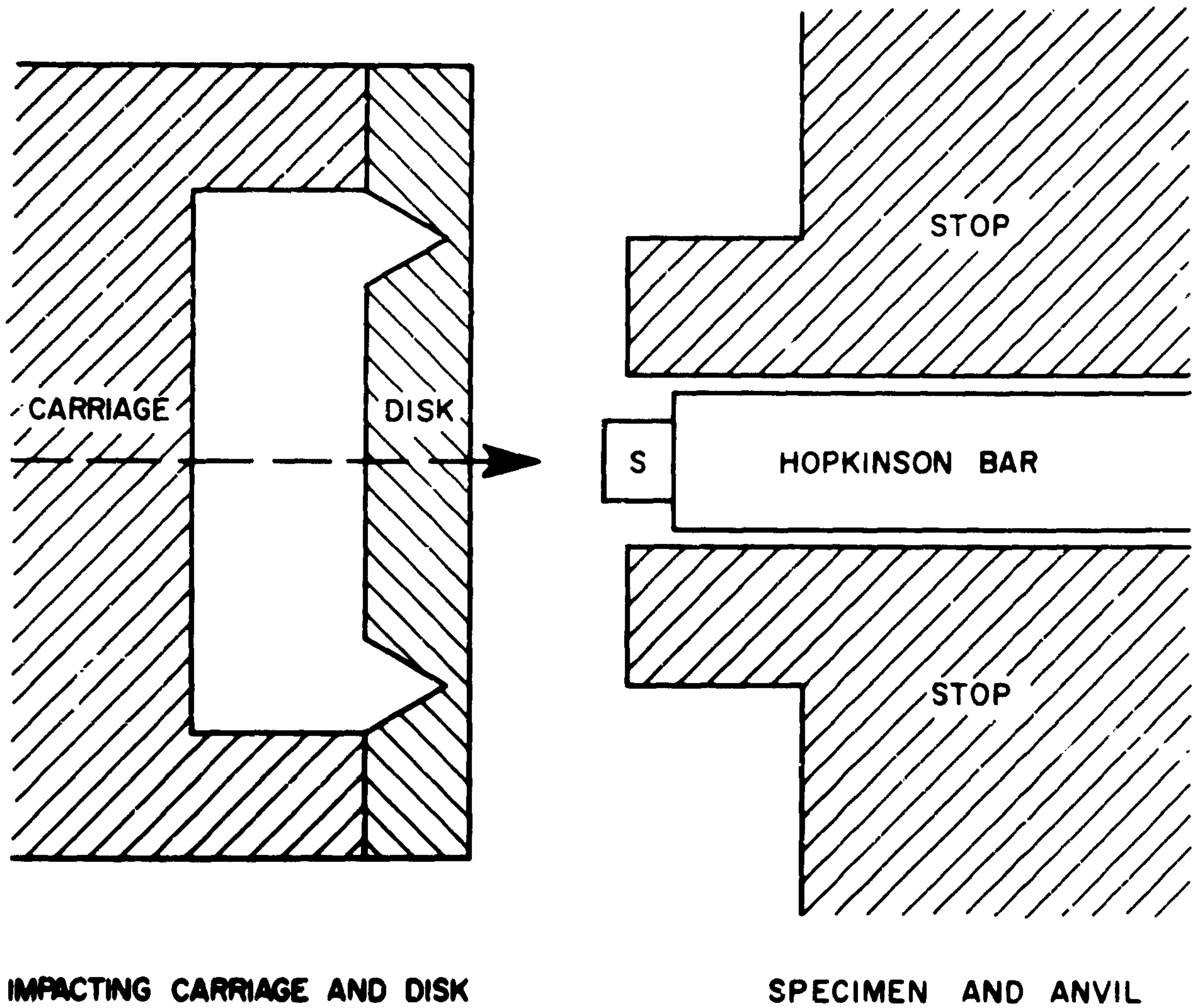

FIG. 1 DYNAMIC COMPRESSION TEST.
S REPRESENTS THE SPECIMEN.

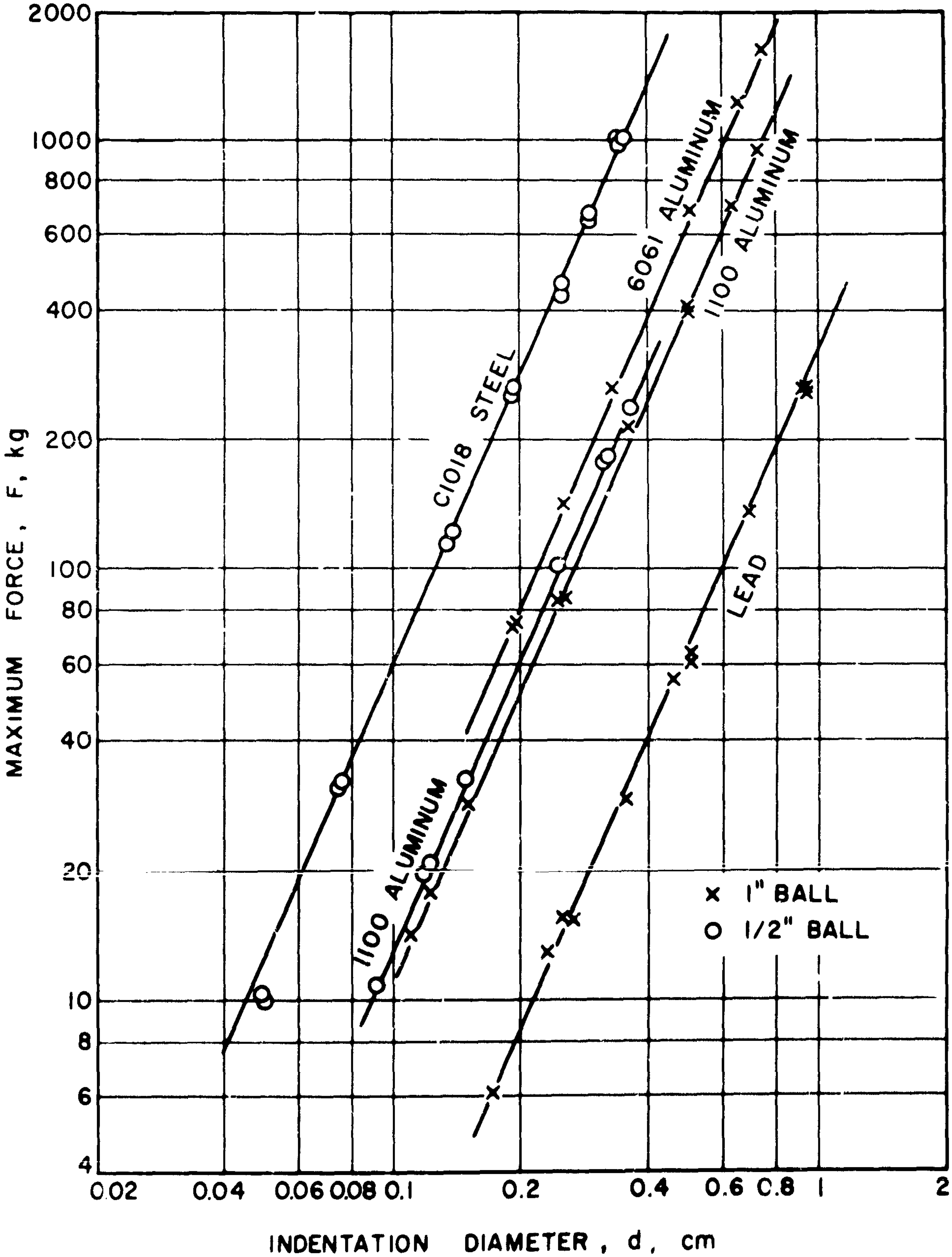

FIG. 2 RESULTS OF STATIC BALL TESTS. MAXIMUM FORCE ATTAINED VERSUS DIAMETER OF INDENTATION. ALL MATERIALS ARE IN THE ANNEALED CONDITION AS SPECIFIED IN TABLE 1. $d/5tD \approx 0.001$ sec^{-1}.

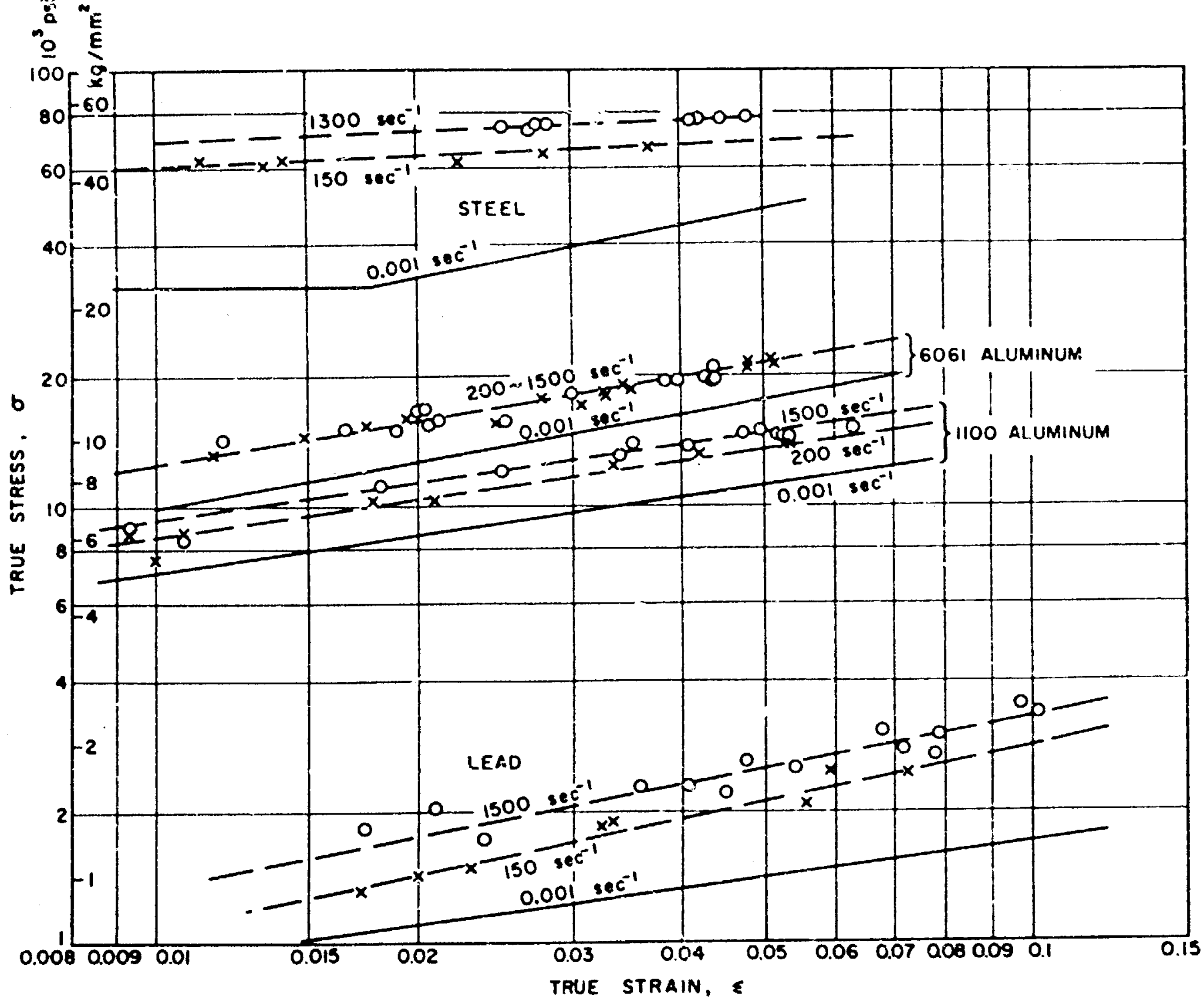

FIG. 3 RESULTS OF SIMPLE COMPRESSION TESTS AT VARIOUS STRAIN RATES. SOLID LINES SHOW RESULTS OF STATIC TESTS. DOTTED LINES ARE BEST CURVES THROUGH THE TEST POINTS. ALL MATERIALS ARE IN THE ANNEALED CONDITION AS SPECIFIED IN TABLE I.

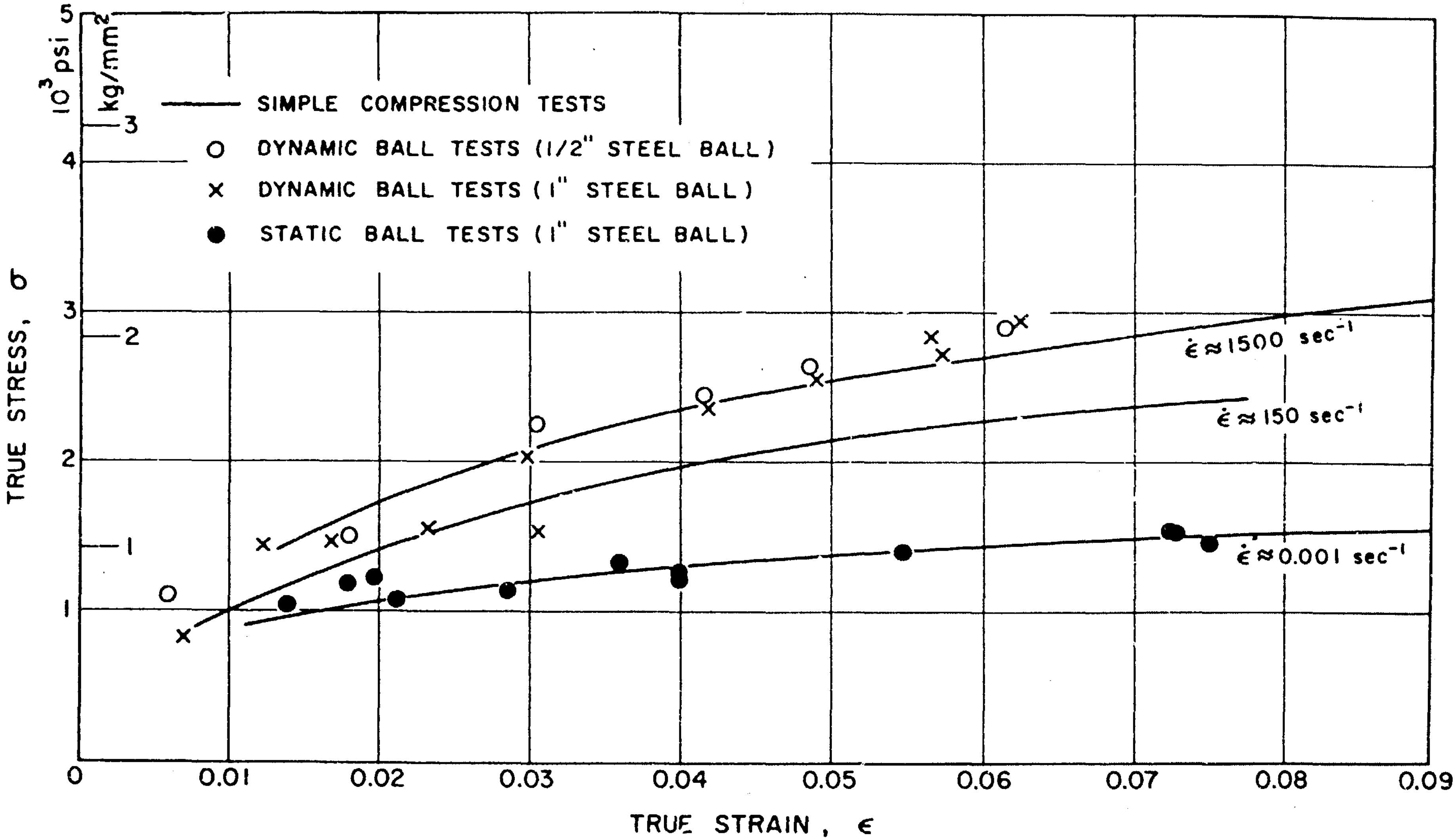

FIG. 4 THE YIELD STRESS OF LEAD UNDER IMPACT. COMPARISON OF RESULTS FROM SIMPLE COMPRESSION TESTS AND BALL TESTS. IN THE BALL TESTS THE YIELD STRESS IS P/3.59 AND THE CORRESPONDING STRAIN IS d/5D.

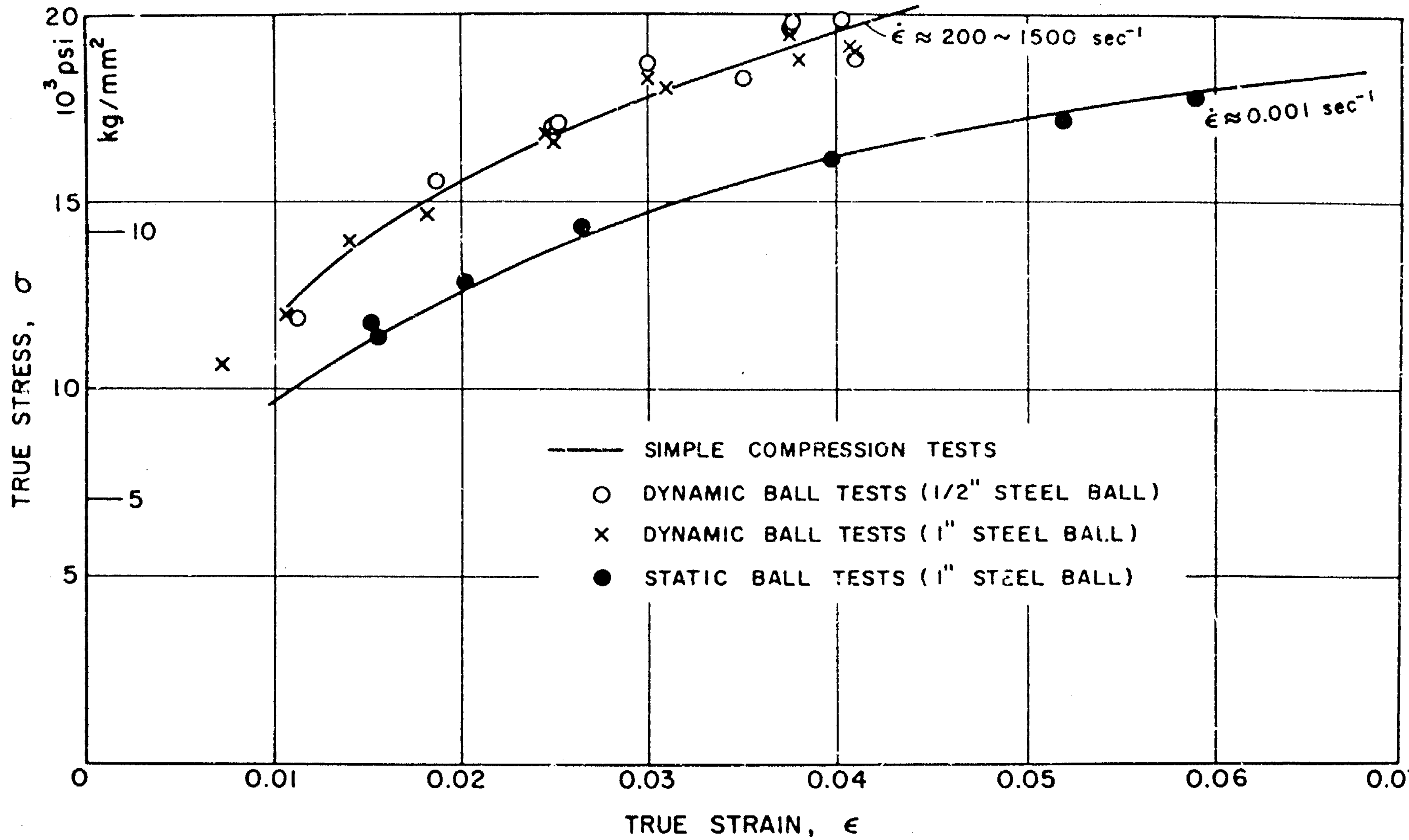

FIG. 5 THE YIELD STRESS OF ANNEALED 6061-T6 ALUMINUM UNDER IMPACT. COMPARISON OF RESULTS FROM SIMPLE COMPRESSION TESTS AND BALL TESTS. IN THE BALL TESTS THE YIELD STRESS IS P/3.02 AND THE CORRESPONDING STRAIN IS d/5D.

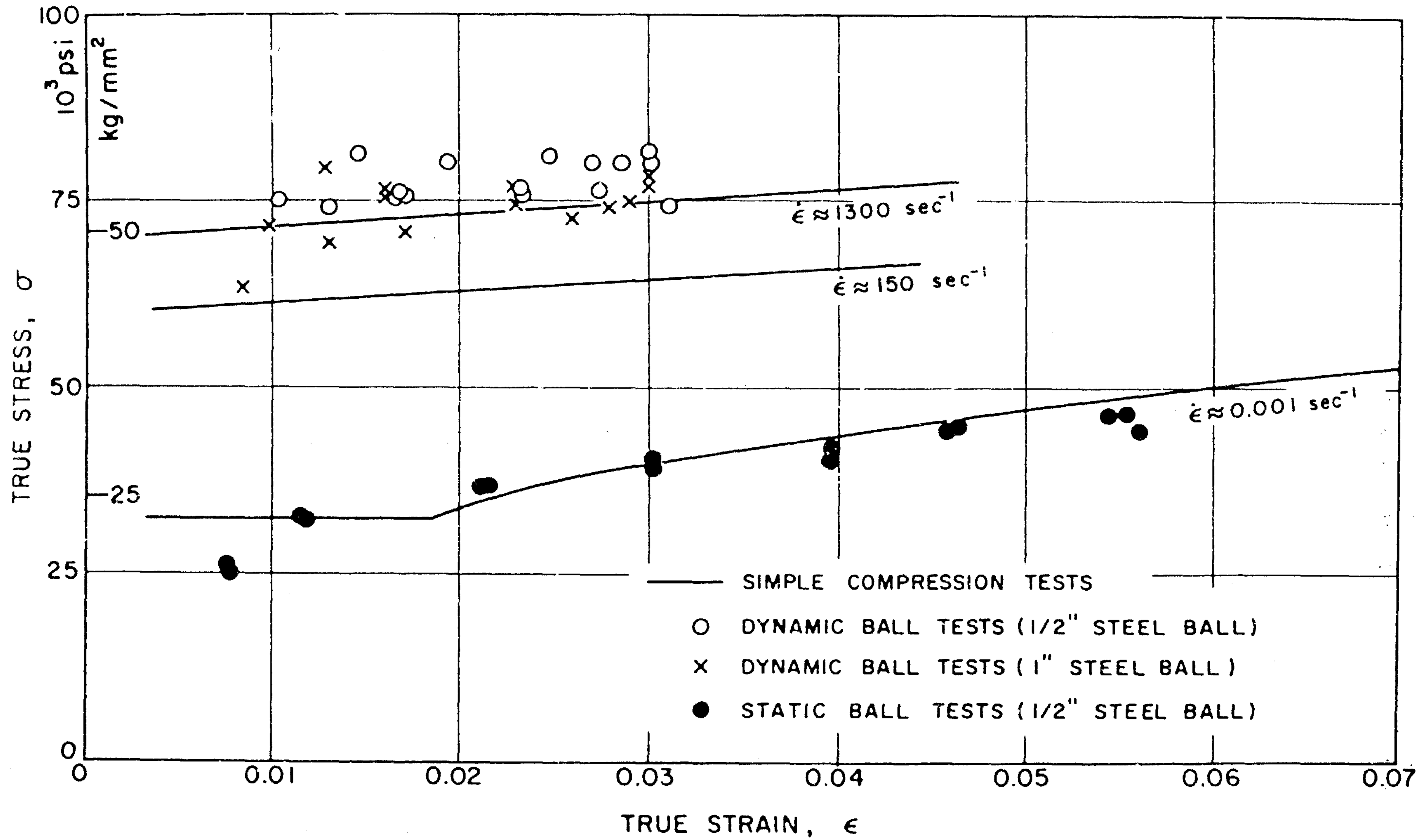

FIG. 6 THE YIELD STRESS OF ANNEALED C1018 STEEL UNDER IMPACT. COMPARISON OF RESULTS FROM SIMPLE COMPRESSION TESTS AND BALL TESTS. IN THE BALL TESTS THE YIELD STRESS IS P/3.16 AND THE CORRESPONDING STRAIN IS d/5D.

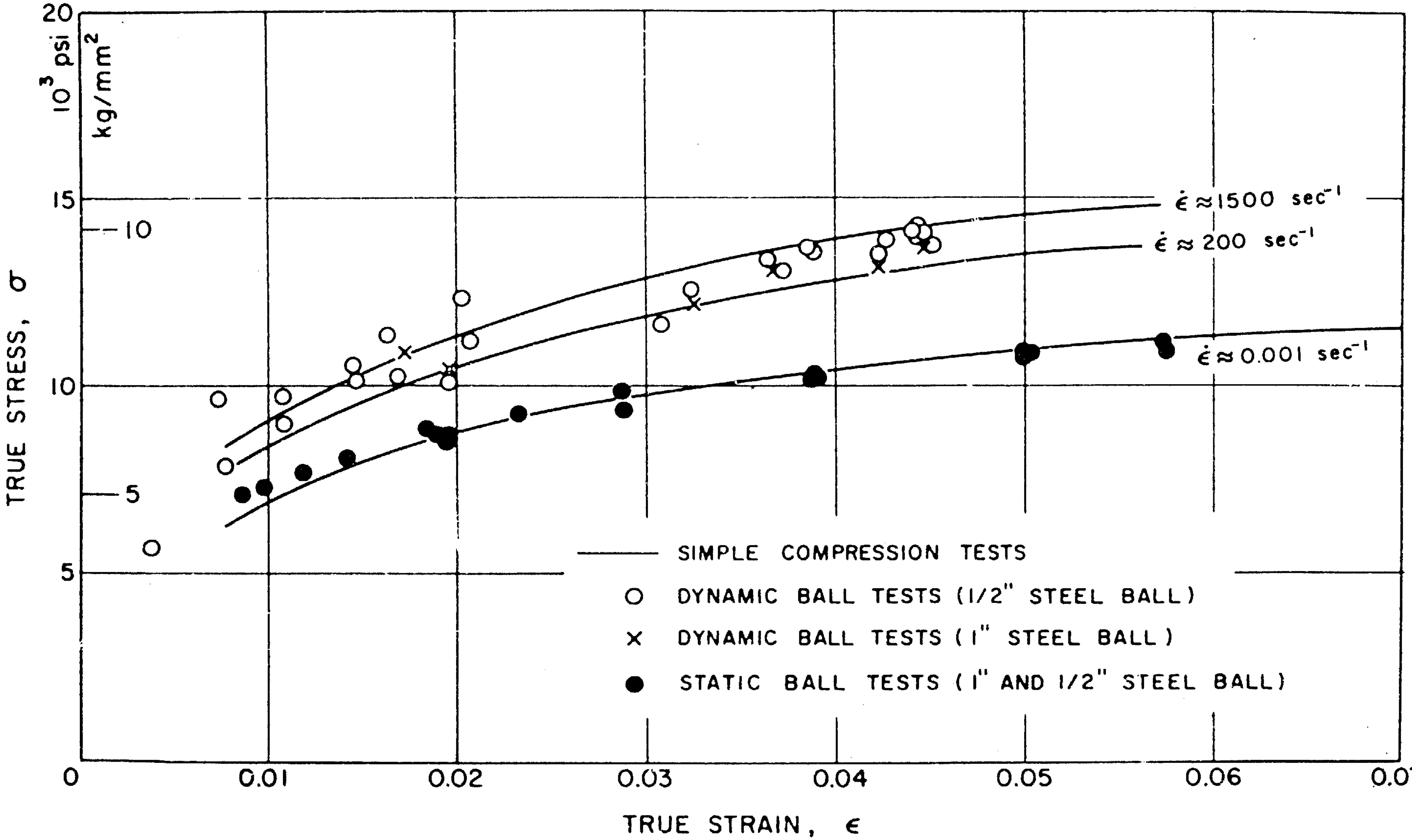

FIG. 7 THE YIELD STRESS OF ANNEALED 1100F. ALUMINUM UNDER IMPACT. COMPARISON OF RESULTS FROM SIMPLE COMPRESSION TESTS AND BALL TESTS. IN THE BALL TESTS THE YIELD STRESS IS P/2.95 AND THE CORRESPONDING STRAIN IS d/5D.

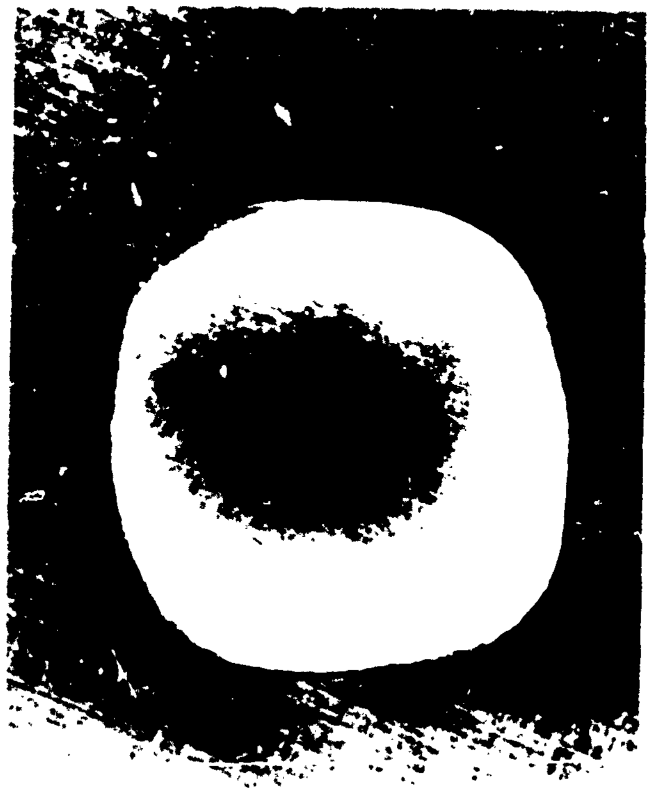

Figure 8. Permanent Indentation in Lead Specimen showing Alternate Regions of "Piling-Up" and "Sinking-In" Around Circumference. Darker Areas Represent "Piling-Up". Approximate width of indentation along a diagonal is 11 mm.

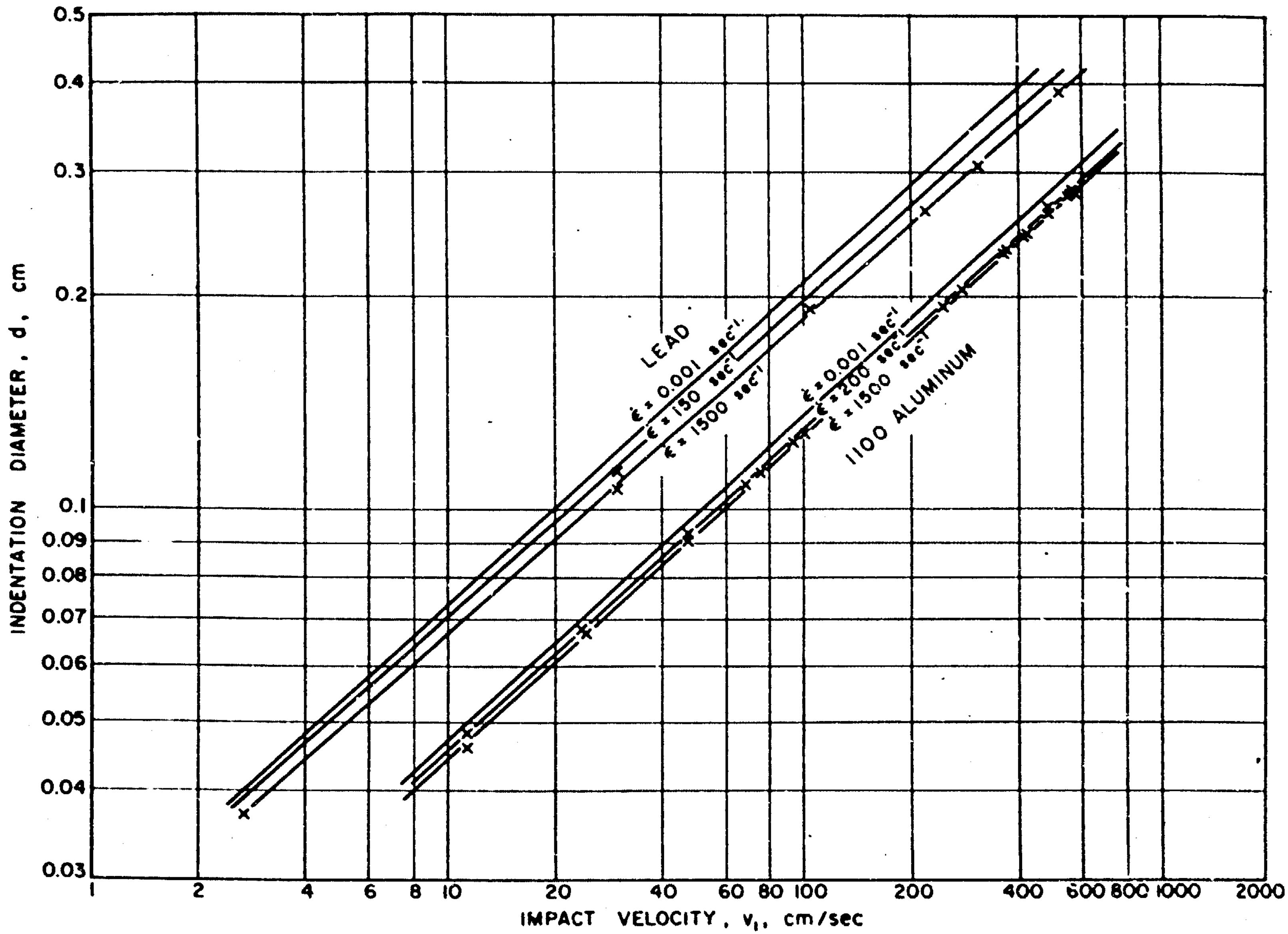

FIG. 9 DIAMETER OF INDENTATION FOR A 1/2" BALL STRIKING LEAD AND ANNEALED 1100F ALUMINUM. POINTS REPRESENT TEST RESULTS. LINES REPRESENT PREDICTED RESULTS USING Eq. (10) AND THE STRESS – STRAIN CURVE CORRESPONDING TO THE APPROPRIATE STRAIN-RATE.

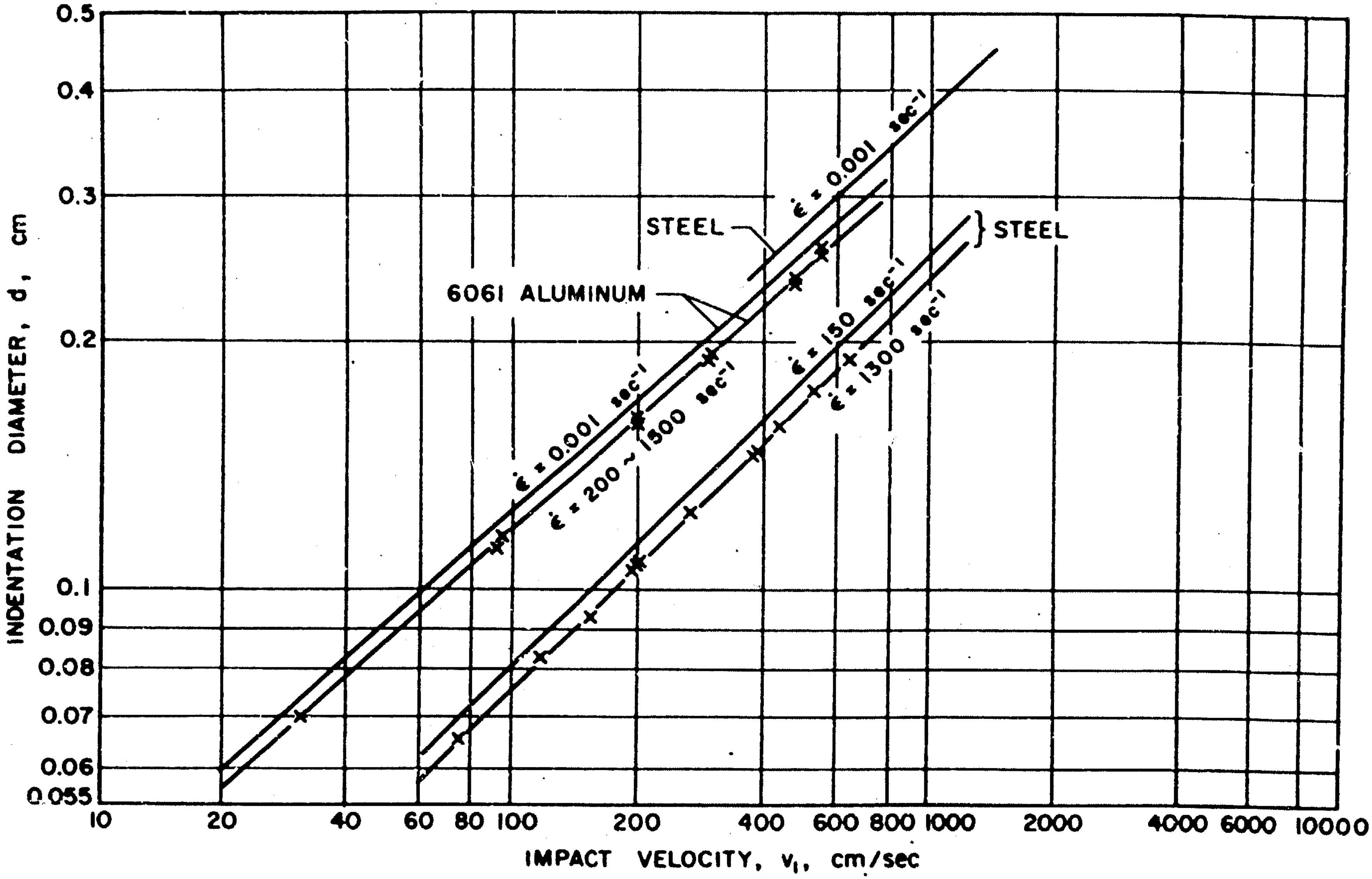

FIG. 10 DIAMETER OF INDENTATION FOR A 1/2" STEEL BALL STRIKING ANNEALED 6061-T6 ALUMINUM AND ANNEALED C1018 STEEL. POINTS REPRESENT TEST RESULTS. LINES REPRESENT PREDICTED RESULTS USING Eq.(10) AND THE STRESS-STRAIN CURVE CORRESPONDING TO THE APPROPRIATE STRAIN-RATE.

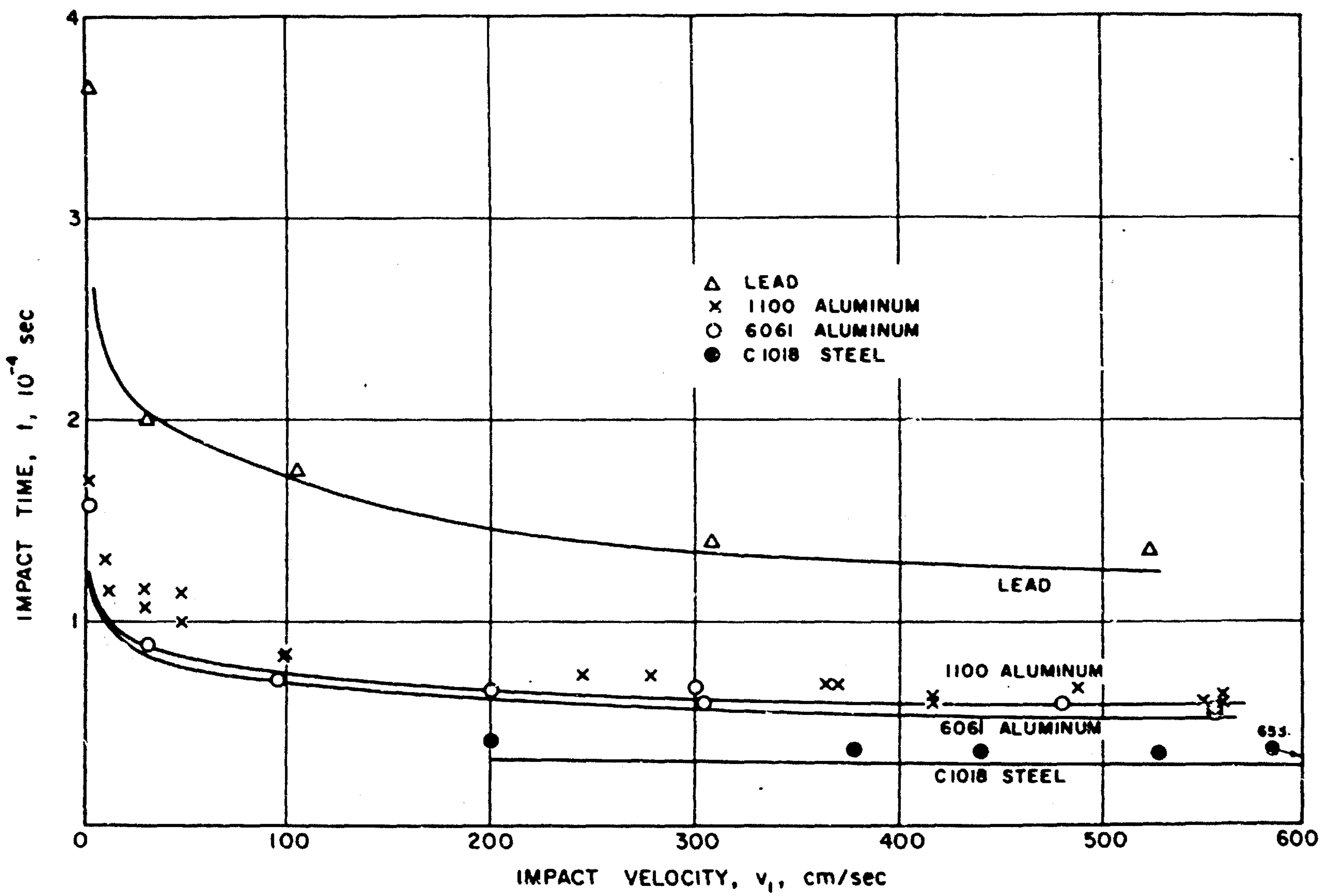

FIG. 11 TIME OF CONTACT DURING IMPACT WITH 1/2" STEEL BALL. LINES SHOW RESULTS COMPUTED USING Eq. (14) AND THE STRESS-STRAIN CURVE CORRESPONDING TO A STRAIN-RATE OF 1500 sec^{-1} (FIG. 3). POINTS SHOW TEST RESULTS. ALL MATERIALS ARE IN THE ANNEALED CONDITION AS SPECIFIED IN TABLE I.

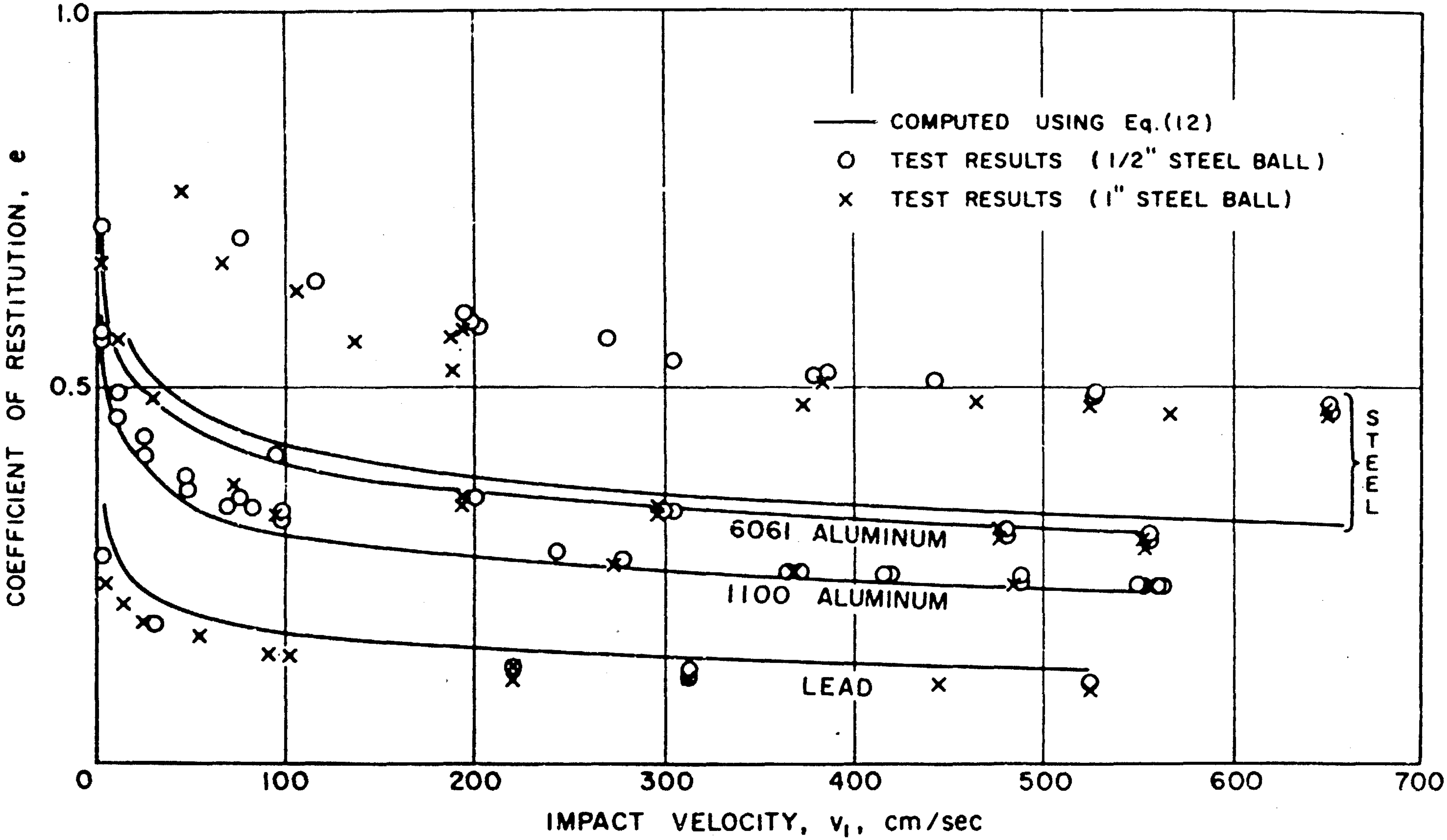

FIG. 12 COEFFICIENT OF RESTITUTION FOR IMPACT WITH 1" AND 1/2" STEEL BALLS. ALL MATERIALS ARE IN THE ANNEALED CONDITION AS SPECIFIED IN TABLE I.

APPENDIX A

Review of Previous Work

In one of the early impact hardness experiments, Martel [1] showed over a wide range of experimental conditions the volume of the indentation formed by the indenter striking the specimen surface was directly proporti to its kinetic energy. On the basis of this observation he suggested that throughout the impact process the indenter was resisted by the specimen constant yield pressure which may be calculated from the total impact energ the indenter and the volume of indentation. For a ball of diameter D, mass m, striking with a velocity v_1 to form an indentation of diameter d on specimen surface, the constant yield pressure is given as

$$(P)_M = \frac{1}{2} mv_1^2/(\pi d^4/32D) \tag{A1}$$

according to Martel. In the investigation of plastic impact problems, later workers (e.g. Andrews [2 to 4] and Tabor [5]) tried to perfect Martel's mode including the elastic properties of the material and found some agreement be the experimental values of the coefficient of restitution, diameter of inden tion, impact time and their theoretical predictions. Tabor also suggested in the calculation of a mean yield pressure, the rebound energy should be su tracted from the total impact energy and that the elastic recovery within th indentation should be taken into account. He obtained

$$(P)_T = (1 - \frac{3}{8} e^2)(\frac{1}{2} mv_1^2)/(\pi d^4/32D) \tag{A2}$$

where e is the coefficient of restitution defined as the ratio of rebound velocity to impact velocity. Furthermore, looking at only the rebound proce

and treating the recovery stage as solely elastic and obeying Hertzian theory for the impact of elastic bodies, Tabor obtained the yield pressure at the beginning of the recovery stage

$$P_r^2 = \frac{80}{3} \left(\frac{1}{2} mv_2^2\right)/\pi^2 d^3 K^2 \qquad \text{(A3)}$$

where v_2 is the rebound velocity, K is the function $[(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2]^{1/2}$ with ν_1, E_1 as the Poisson's ratio and Young's modulus of the ball and ν_2, E_2 as those of the specimen. Tabor found experimentally that P_r always has a value less than $(P)_T$ and closer to 1.1 P_s, where P_s is the yield pressure needed to produce an indentation of the same size in a static test. Observing that the difference between $(P)_T$ and P_r is greater for softer materials, Tabor concluded that it was associated with an apparent viscous behavior of the material. Since the acceleration of the material around the indentation must be much greater at the beginning and at the middle of the impact stage than at the end of the impact, the mean yield pressure $(P)_T$ over the whole impact should have a value greater than P_r due to the viscous behavior of the material.

Using a piezo-electric crystal, Crook [6] conducted tests to measure the force throughout the impact of two colliding bodies at impact velocities ranging from 10 cm/sec to 100 cm/sec. He found that the approximation of a constant yield pressure was valid up to the instant of maximum impact force. In addition, he found that Tabor's $(P)_T$ predicted rather accurately the magnitude of the yield pressure at the instant of maximum impact force. However, he disagreed with Tabor on the influence of the viscosity of the material on the yield pressure; he pointed out that the yield pressure would not remain substantially constant over a large portion of the impact if it were influenced by the viscosity of the material.

The concept of a constant yield pressure, however, seems to be in(sistent with the fact that for most materials the yield stress has been f(depend on strain and strain-rate. Recently, Goldsmith and Lyman [7] meası the force of impact at velocities ranging from 750 cm/sec to 4,500 cm/sec found that yield pressure did not remain constant throughout the impact. also found that for some materials a higher yield pressure was obtained dynamic than under static conditions. In another paper with Yew [8], Gol again indicated the dependence of yield pressure on strain and strain- measuring the internal strain distribution produced in a lead specimen by impact of a ball.

In a previous paper [9], the authors pointed out that the yield calculated with Tabor's formula, Equation (A2), was dependent on the impa velocity and presumably the resulting strain and strain-rate. In additi of impact velocity against indentation diameter on a log-log coordinate showed slopes different from the 0.5 value based on constant yield pressu concept.

Experiments with conical indenters [10] also indicated that the yi pressure was strain-rate dependent. Using the results of dynamic and stat indentation tests with a quasi-static method of analysis and the constant pressure assumption, Davis and Hunter [11 and 12] obtained a ratio of dyna static yield pressures. This ratio was found to agree approximately with ratio of dynamic and static yield stresses obtained by other investigators the same materials in simple compression tests.

At the present time, no rigorous theoretical solution exists for indentation problem, even for the idealized rigid-perfectly plastic materi Ishlinsky, however, was able to obtain a theoretical solution using the Ha Karman yield condition and the slip line method [13]. He found that the m

yield pressure was nearly independent of the size of the indentation and equal to $c\sigma$, where c is a constant with a value close to 2.66 and σ is the yield stress of the rigid-perfectly plastic material. Theoretical studies for various rigid indenters on rigid-perfectly plastic materials also gave values of c between 2.6 and 3 [14 and 15].

Experimentally, Tabor found that c was close to 2.8 for highly worked metals when indented statically with a ball [5]. Furthermore, by relating linearly the indentation diameter to the strain Tabor extended this result to cover work-hardening materials, so that

$$P = c\sigma \tag{A4}$$

$$\varepsilon = d/5D \tag{A5}$$

where ε is the strain corresponding to the yield stress σ , d the indentation diameter, D the ball diameter, and c a constant having a value close to 2.8 for two metals. He found the above formulae empirically by comparing the results of frictionless simple compression tests with those of ball indentation tests. The comparisons were made by matching the results of compression tests with the results of indentation tests on a basis of Vickers hardness numbers. He found that consistent results could be obtained if he chose the Vickers hardness value taken at the edge of the indentation as the characteristic hardness of an indentation.

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APPENDIX B

Analysis of the Plastic Work-Hardening Stage

As pointed out in the text, a two-stage model of the dynamic indentat test is employed to predict the indentation diameter, the total time of cont and the coefficient of restitution. This model consists of a plastic work-hardening stage followed by an elastic recovery stage. The ball strikes the specimen at the start of the plastic stage with an initial velocity v_1. Its subsequent motion during penetration is resisted by a yield pressure, P dependent on the strain (i.e. dependent on the diameter of the contact area, Using Tabor's empirical formulae, Equations (1) and (2) and a stress-strain tion of the form $\sigma = b\varepsilon^n$, it can be shown that

$$P = \frac{1}{5} cb\,(a/D)^n \qquad \text{(B.}$$

where D is the diameter of the ball and c is Tabor's constant. For the motion of the ball, Newton's law gives

$$-\frac{\pi a^2}{4} P = m\ddot{x} \qquad \text{(B2}$$

where m is the mass of the ball and $\ddot{x}$ the second time derivative of the relative approach of the mass centers of the two impinging bodies. If ther not much "piling up" or "sinking in" near the contact area and the ball is then the equation

$$a^2 = 4\,D\,x \qquad \text{(B3}$$

holds approximately.

Using Equations (B1), (B2) and (B3), it can be shown that at the moment of maximum relative approach (or of zero velocity of approach) the diameter of the area of contact is

$$d = f_1(c,n,b) \quad D \quad (mv_1^2/D^3)^{1/(4+n)} \qquad \text{(B4)}$$

where

$$f_1(c,n,b) = [5^n \cdot 4 \cdot (4 + n)/\pi cb]^{1/(4+n)} .$$

when the plastic deformation is pronounced and the elastic recovery is relatively small, this diameter can be considered to be approximately equal to that of the permanent indentation.

Replacing a with d and using Equation (B4) to find the product cb, Equation (B1) gives the yield pressure at the maximum approach, P, as

$$P = \frac{4 + n}{4} \left(\frac{1}{2} mv_1^2 \right)/(\pi d^4/32) . \qquad \text{(B5)}$$

As pointed out in the text (Section II), this expression for the yield pressure is the same as that derived from the impact test results.

Equations (B1), (B2) and (B3) can be used to obtain also the duration of the plastic strain-hardening stage (the time lapse between the instant when the impact starts and the instant when the maximum relative approach is attained), t_p, so that

$$t_p = I\frac{x_o}{v_1} = \frac{I}{4} [f_1(c,n,b)]^2 (D/v_1)(mv_1^2/D^3)^{2/(4+n)} \qquad \text{(B6)}$$

where x_o is the maximum relative approach between the two bodies and I is the convergent series

$$1 + \frac{1}{6 + n} + \frac{1 \cdot 3}{2 \cdot 4 \cdot (5 + n)} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot (7 + 3n/2)} + \ldots .$$

The value of I changes only from $\pi/2$ to 1.47 for a change of n from 0 tc 1. As a result, for values of n between o and 1

$$t_p = \frac{\pi}{8}\,[f_1(c,n,b)]^2 \;(D/v_1)(mv_1^2/D^3)^{2/(4+n)} \quad . \qquad \text{(B7)}$$

APPENDIX C

Results of Static and Dynamic Indentation Tests

TABLE C1

Results of Static Ball Tests on Lead using a 1" Steel Ball

$$\frac{d}{5tD} = 0.001 \text{ sec}^{-1}$$

Maximum Force, F (kg)	Indentation Diameter, d (cm)
6.12	0.172
12.9	0.233
15.6	0.253
15.4	0.268
29.2	0.360
56.0	0.463
63.5	0.505
61.9	0.507
136	0.697
265	0.921
266	0.929
260	0.948

TABLE C2

Results of Static Ball Tests on Annealed 6061-T6 Aluminum using a 1" Steel Ball

$$\frac{d}{5tD} = 0.001 \text{ sec}^{-1}$$

Maximum Force, F (kg)	Indentation Diameter, d (cm)
74.3	0.193
74.8	0.198
142	0.257
265	0.335
686	0.506
1230	0.660
1637	0.749

TABLE C3

Results of Static Ball Tests on Annealed C1018

Steel using a 1/2" Steel Ball

$$\frac{d}{5tD} = 0.001 \text{ sec}^{-1}$$

Maximum Force, F (kg)	Indentation Diameter, d (cm)
10.1	0.0469
10.4	0.0489
31.4	0.0746
32.2	0.0758
113	0.133
123	0.138
262	0.192
253	0.191
440	0.251
466	0.251
680	0.294
656	0.291
976	0.346
1010	0.357
1000	0.351

TABLE C4a

Results of Static Ball Tests on Annealed 1100F Aluminum using a 1" Steel Ball

$$\frac{d}{5tD} = 0.001 \text{ sec}^{-1}$$

Maximum Force, F (kg)	Indentation Diameter, d (cm)
14.3	0.111
18.0	0.123
28.8	0.151
84.9	0.245
84.9	0.247
215	0.365
409	0.498
406	0.493
702	0.633
953	0.730

TABLE C4b

Results of Static Ball Tests on Annealed 1100F Aluminum using a 1/2" Steel Ball

$$\frac{d}{5tD} = 0.001 \text{ sec}^{-1}$$

Maximum Force, F (kg)	Indentation Diameter, d (cm)
10.9	0.0911
20.1	0.118
21.1	0.122
33.0	0.148
51.6	0.183
103	0.247
180	0.318
183	0.321
[illegible]	0.363

TABLE C5

Results obtained with a 1" Steel Ball striking Lead

Impact Velocity v_1 (cm/sec)	Impact Height h_1 (cm)	Rebound Height h_2 (cm)	Coeff. of Restitution e	Impact Time, t (10^{-4} sec)	Indentation Diameter d (cm)
3.5			0.240	6.62	0.088
13.9			0.210	4.08	0.156
26.2			0.186	3.85	0.213
54.0			0.169	3.44	0.296
91.7			0.144	3.54	0.388
102			0.142	(10.14)	0.380
221	25.0	0.31	0.111	--	0.531
221	25.0	0.40	0.126	2.94	0.530
313	50.0	0.68	0.117	2.88	0.622
443	100.2	1.14	0.107	--	0.720
443	100.2	1.17	0.108	2.74	0.728
523	139.6	1.53	0.105	2.77	0.794

TABLE C6

Results obtained with a 1/2" Steel Ball Striking Lead

Impact Velocity v_1 (cm/sec)	Impact Height h_1 (cm)	Rebound Height h_2 (cm)	Coeff. of Restitution e	Impact Time, t (10^{-4} sec)	Indentation Diameter d (cm)
2.67			0.272	3.66	0.037
30.2			0.182	2.00	0.115
105			(0.087)	1.75	0.194
221	25	0.39	0.125	--	0.264
313	50	0.79	0.126	1.40	0.308
313	50	0.70	0.118	--	0.313
313	50	0.70	0.118	--	0.313
524	140	1.70	0.110	1.37	0.390

TABLE C7

Results obtained with a 1" Steel Ball striking Annealed 6061-T6 Aluminum

Impact Velocity v_1 (cm/sec)	Impact Height h_1 (cm)	Rebound Height h_2 (cm)	Coeff. of Restitution e	Impact Time, t (10^{-4} sec)	Indentation Diameter d (cm)
2.1			0.665	3.28	--
12.5			0.563	2.44	0.091
29.9			0.490	1.74	0.135
54.9			0.383	1.55	0.178
94.6			0.328	1.45	0.231
194	19.3	2.28	0.344	1.41	0.317
194	19.3	2.35	0.351	1.37	0.316
295	44.4	5.11	0.339	1.31	0.381
295	44.4	5.[illegible]4	0.340	1.29	0.382
476	116	11.1	0.310	1.24	0.478
476	116	10.4	0.299	1.09	0.481
554	157	13.2	0.291	1.14	0.518
554	157	13.2	0.291	1.15	0.519

TABLE C8

Results obtained with a 1/2" Steel Ball

striking Annealed 6061-T6 Aluminum

Impact Velocity v_1 (cm/sec)	Impact Height h_1 (cm)	Rebound Height h_2 (cm)	Coeff. of Restitution e	Impact Time, t (10^{-4} sec)	Indentation Diameter d (cm)
2.01			0.717	1.58	--
30.2			0.426	0.89	0.071
94.7			0.41	0.72	0.118
200	20.5	--	--	0.67	0.160
200	20.5	2.62	0.357	0.67	0.160
304	47.2	5.40	0.338	0.59	0.194
299	45.6	5.07	0.333	0.69	0.191
480	117	11.0	0.306	0.61	0.240
480	117	11.2	0.309	--	0.239
556	158	14.0	0.298	0.55	0.260
556	158	14.6	0.304	0.54	0.257

TABLE C9

Results obtained with a 1" Steel Ball striking Annealed C1018 Steel

Impact Velocity v_1 (cm/sec)	Impact Height h_1 (cm)	Rebound Height h_2 (cm)	Coeff. of Restitution e	Impact Time, t (10^{-4} sec)	Indentation Diameter d (cm)
136	9.52	2.97	0.559	0.91	0.175
187	17.9	5.79	0.569	--	0.211
106	5.79*	2.28	0.628	--	0.160
66.8	2.28*	1.01	0.666	--	0.125
44.5	1.01*	0.587	0.762	--	0.106
372	70.8	18.3	0.476	0.72	0.294
189	18.3 *	5.99	0.522	--	0.209
383	74.9	19.2	0.506	--	0.296
194	19.2 *	6.32	0.574	--	0.211
463	110	25.3	0.480	0.72	0.330
525	141	31.4	0.472	0.70	0.350
566	164	35.0	0.462	--	0.362
650	216	44.8	0.456	0.68	0.384
650	216	46.9	0.466	0.68	0.385

* Successive bounces in a single test.

TABLE C10

Results obtained with a 1/2" Steel Ball striking Annealed C1018 Steel

Impact Velocity v_1 (cm/sec)	Impact Height h_1 (cm)	Rebound Height h_2 (cm)	Coeff. of Restitution e	Impact Time, t (10^{-4} sec)	Indentation Diameter d (cm)
199	20.2	6.96	0.587	0.41	0.107
117	6.96*	2.85	0.640	--	0.0826
74.7	2.85*	1.39	0.699	--	0.0658
378	72.9	19.5	0.516	0.37	0.147
195	19.5*	6.99	0.599	--	0.106
386	76.1	20.7	0.521	0.37	0.148
202	20.7	6.96	0.579	--	0.108
441	99.4	25.7	0.509	0.35	0.157
270	37.1	11.9	0.565	--	0.123
152	11.9	--	--	--	0.092
527	142	34.5	0.493	0.34	0.172
527	142	34.8	0.495	0.35	0.174
651	217	47.7	0.469	--	0.194
651	217	47.1	0.477	--	0.190
651	217	47.6	0.469	0.33	0.191
305	47.6*	13.7	0.536	--	0.131

* Successive bounces in a single test.

TABLE C11

Results obtained with a 1" Steel Ball striking Annealed 1100F Aluminum

Impact Velocity v_1 (cm/sec)	Impact Height h_1 (cm)	Rebound Height h_2 (cm)	Coeff. of Restitution e	Impact Time, t (10^{-4} sec)	Indentation Diameter d (cm)
274	38.3	2.64	0.262	1.45	0.412
71.9	2.64*	0.36	0.369	1.54	0.217
367	68.6	4.27	0.250	1.44	0.469
91.6	4.27*	0.471	0.332	--	0.248
484	119	6.69	0.237	1.37	0.537
556	158	8.37	0.230	1.32	0.570

* Successive bounces in a single test.

TABLE C12

Results obtained with a 1/2" Steel Ball striking Annealed 1100F Aluminum

Impact Velocity v_1 (cm/sec)	Impact Height h_1 (cm)	Rebound Height h_2 (cm)	Coeff. of Restitution e	Impact Time, t (10^{-4} sec)	Indentation Diameter d (cm)
2.26			0.561	---	0.0236
3.73			0.574	1.70	0.0245
11.2			0.494	1.31	0.0460
11.3			0.460	1.15	0.0485
23.8			0.435	1.17	0.0684
24.5			0.410	1.07	0.0679
46.6			0.380	1.14	0.0926
47.2			0.362	1.00	0.0923
98.4			0.334	0.83	0.131
98.9			0.322	0.83	0.128
244	30.3	2.39	0.282	0.74	0.196
68.4	2.39*	0.273	0.338	--	0.107
278	39.5	2.93	0.272	0.74	0.205
75.7	2.93*	0.367	0.354	--	0.110
364	67.4	4.32	0.253	0.70	0.231
92.1	4.32*	0.496	0.339	--	0.125
370	69.9	4.51	0.254	0.70	0.235
416	88.4	5.55	0.251	0.63	0.246
416	88.4	5.55	0.251	0.62	0.246
487	121	7.05	0.241	0.68	0.268
487	121	7.21	0.244	0.68	0.265
550	155	8.66	0.236	--	0.280
559	155	8.54	0.235	0.60	0.280
559	159	8.71	0.234	0.60	0.286
	159	8.69	0.233	0.64	0.283

* Successive bounces in a single test.